Solutions, Tutorial 6, 2014

Solutions, Examination EF2240, 2010-10-21

1.

a)

I take the following values for day 131:

V_R = 250 km s^{-1} = $2.5 \cdot 10^5$ m s^{-1}
N_P = 10^{-1} cm^{-3} = 10^5 m-3

The standoff distance is given by

$$N_p m_p V_R^2 = \frac{1}{2\mu_0}\left(\frac{\mu_0 a}{4\pi r^3}\right)^2 \quad \Rightarrow$$

$$r = \left(\frac{\mu_0 a^2}{32\pi^2 N_p m_p V_R^2}\right)^{1/6}$$

<u>For day 131:</u>

$$N_p m_p V_R^2 = \frac{1}{2\mu_0}\left(\frac{\mu_0 a}{4\pi r^3}\right)^2 \quad \Rightarrow$$

$$r = \left(\frac{4\pi \cdot 10^{-7}\left(8 \cdot 10^{22}\right)^2}{32\pi^2 \cdot 10^5 \cdot 1.67 \cdot 10^{-27} \cdot \left(2.5 \cdot 10^5\right)^2}\right)^{1/6} = 1.16 \cdot 10^8 \text{ m} = 18.2 \text{ R}_\text{E}$$

<u>For day 130:</u>

V_R = $4.0 \cdot 10^5$ m s^{-1}
N_P = $4 \cdot 10^6$ m-3

$$r = \left(\frac{4\pi \cdot 10^{-7}\left(8 \cdot 10^{22}\right)^2}{32\pi^2 \cdot 4 \cdot 10^5 \cdot 1.67 \cdot 10^{-27} \cdot \left(4 \cdot 10^5\right)^2}\right)^{1/6} = 5.4 \cdot 10^7 \text{ m} = 8.4 \text{ R}_\text{E}$$

The ratio between the stand-off distance is 2.2

b)

$$\Psi = \arctan\left(\frac{\omega_{sun} r}{V_R}\right)$$

$$\omega_{sun} = \frac{2\pi}{25 \cdot 24 \cdot 3600} = 2.91 \cdot 10^{-6}$$

r = 1 AU

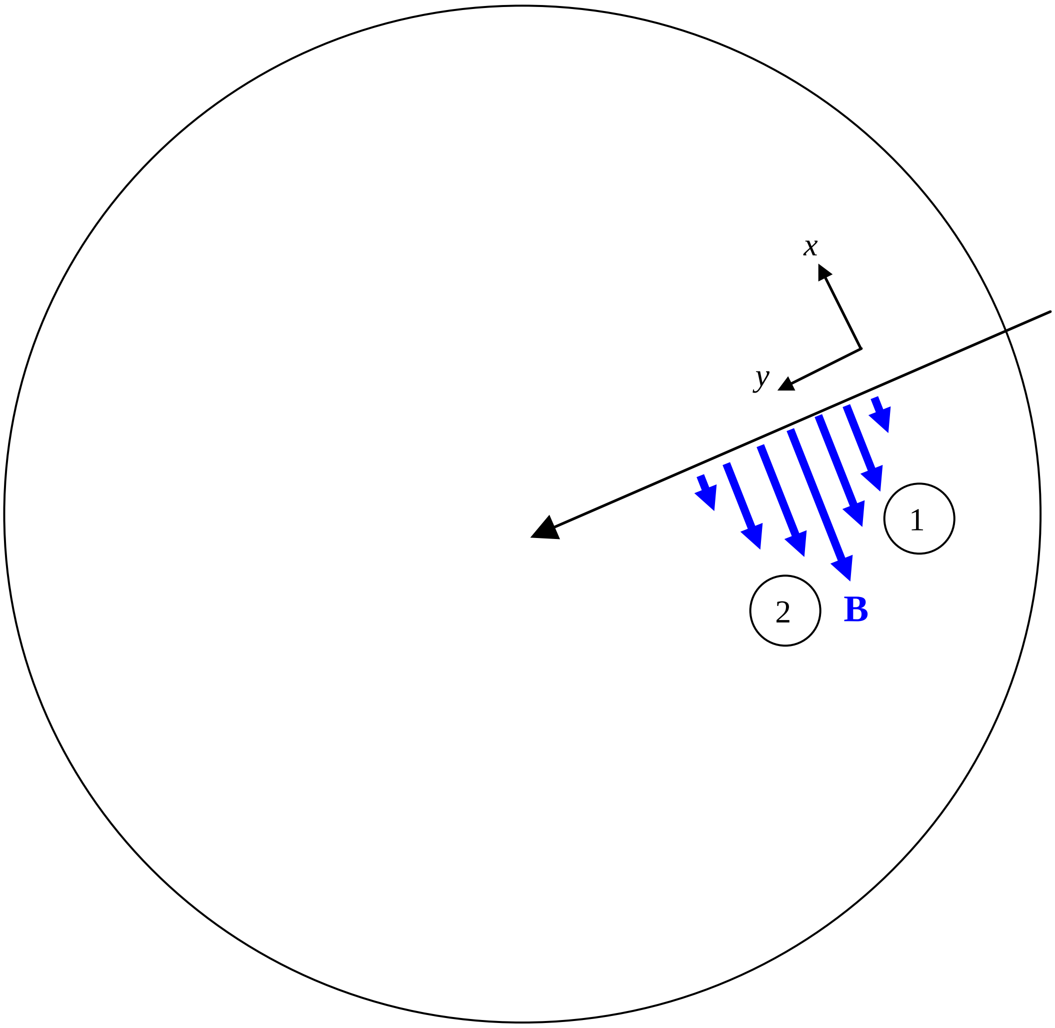
Then

$$\Psi = \arctan\left(\frac{4.36 \cdot 10^5}{V_R}\right)$$

$$\Psi_{\min} = \arctan\left(\frac{4.36 \cdot 10^5}{5 \cdot 10^5}\right) = 42.6^\circ$$

$$\Psi_{\max} = \arctan\left(\frac{4.36 \cdot 10^5}{2.5 \cdot 10^5}\right) = 61.5^\circ$$

2.



$$j_z = -\frac{1}{\mu_0}\frac{\partial B_y}{\partial x}$$

<u>Current sheet 1:</u>

$\frac{\partial B_y}{\partial x} < 0 \quad \Rightarrow \quad j_z > 0$ which means it is an upward current, which is consistent with the statistical result.

$$\Delta B_y \approx \frac{15\,\text{mm}}{22\,\text{mm}} \cdot 1000 \cdot 10^{-9} = 6.8 \cdot 10^{-7}\,\text{T}$$

$$\Delta x \approx \frac{10\,\text{mm}}{10\,\text{mm}} \cdot \frac{2^\circ}{360^\circ} 2\pi\left(R_E + 800\,\text{km}\right) = 250 \cdot 10^3\,\text{m}$$

Then

$$j_z \approx -\frac{1}{\mu_0}\frac{\Delta B_y}{\Delta x} = 2.2 \cdot 10^{-6}\,\text{Am}^{-2}$$

<u>Current sheet 2</u>

$\frac{\partial B_y}{\partial x} > 0 \quad \Rightarrow \quad j_z < 0$ which means it is an downward current, which is consistent with the statistical result.

$$\Delta B_y \approx \frac{18\,\text{mm}}{22\,\text{mm}} \cdot 1000 \cdot 10^{-9} = 8.2 \cdot 10^{-7}\,\text{T}$$

$$\Delta x \approx \frac{10\,\text{mm}}{10\,\text{mm}} \cdot \frac{2^\circ}{360^\circ} 2\pi\left(R_E + 800\,\text{km}\right) = 250 \cdot 10^3\,\text{m}$$

Then

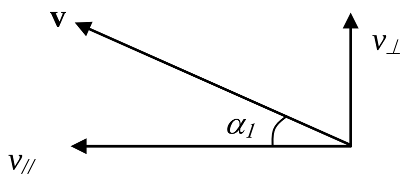
$$j_z \approx -\frac{1}{\mu_0}\frac{\Delta B_y}{\Delta x} = -2.6 \cdot 10^{-6}\,\text{Am}^{-2}$$

3.

a)

$$\alpha_{lc} = \arcsin\sqrt{\frac{B_1}{B_2}} = \arcsin\sqrt{\frac{4000}{50000}} = 16.4^\circ$$

b)



W = $10^3 \cdot 1.6 \cdot 10^{-19}$

$$v = \sqrt{\frac{2W}{m_e}} = \sqrt{\frac{2 \cdot 10^3 \cdot 1.6 \cdot 10^{-19}}{0.91 \cdot 10^{-30}}} = 1.88 \cdot 10^7\,\mathrm{ms^{-1}}$$

$$v_\perp = v \sin\alpha_1 = 6.43 \cdot 10^6\,\mathrm{ms^{-1}}$$

$$v_{//} = v \sin\alpha_1 = 1.77 \cdot 10^7\,\mathrm{ms^{-1}}$$

For the particle to be in the loss cone, we need to increase $v_{//}$ so that

$$\tan\alpha_{lc} = \tan 16.4^\circ = \frac{v_\perp}{v_{//,new}} \quad \Rightarrow$$

$$v_{//,new} = \frac{v_\perp}{\tan 16.4^\circ} = 2.18 \cdot 10^7\,\mathrm{ms^{-1}}$$

Then

$v_{//,new} - v_{//} = 0.41 \cdot 10^7\ \mathrm{ms^{-1}}$.

c)

Thus the extra parallel energy needed is

$$\frac{m_e v_{//,new}^2}{2} - \frac{m_e v_{//}^2}{2} = \frac{0.91 \cdot 10^{-30}}{2}\left(\left(2.18 \cdot 10^7\right)^2 - \left(1.77 \cdot 10^7\right)^2\right) = 7.36 \cdot 10^{-17}\,\mathrm{J} = 460\ \mathrm{eV}$$

which is the energy gained by an electron accelerated by 460 V potential drop, ($W = qV$), which is at the lower end of typical auroral acceleration potentials (typically 0.5-10 kV).

4.

a) Wien's displacement law gives

$$\lambda_{max} = \frac{2.9 \cdot 10^{-3}}{T} = \frac{2.9 \cdot 10^{-3}}{310} = 9.4 \cdot 10^{-6}\text{m} = 9400 \text{ nm} = 9.4 \text{ µm}.$$

This is infra-red radiation.

b)

$$\lambda_{max} = \frac{2.9 \cdot 10^{-3}}{4200} = 6.9 \cdot 10^{-7}\text{m} = 690 \text{ nm}$$

Dark red.

c)

$$P_{sun} = \sigma_{SB} T_{sun}^4 \cdot 4\pi r_{sun}^2$$

$$r_{sun} = \frac{1.39 \cdot 10^9}{2} \text{ m} = 6.95 \cdot 10^8 \text{ m}$$

$$P_{with\,spot} = \sigma_{SB} T_{sun}^4 \cdot \left(4\pi r_{sun}^2 - \pi r_{spot}^2\right) + \sigma_{SB} T_{spot}^4 \pi r_{spot}^2$$

Then

$$\frac{P_{with\,spot}}{P_{sun}} = \frac{\sigma_{SB} T_{sun}^4 \cdot \left(4\pi r_{sun}^2 - \pi r_{spot}^2\right) + \sigma_{SB} T_{spot}^4 \pi r_{spot}^2}{\sigma_{SB} T_{sun}^4 \cdot 4\pi r_{sun}^2}$$
$$= \frac{T_{sun}^4 \cdot \left(4 r_{sun}^2 - r_{spot}^2\right) + T_{spot}^4 r_{spot}^2}{T_{sun}^4 \cdot 4 r_{sun}^2}$$
$$= \frac{6000^4 \cdot (4 \cdot (6.95 \cdot 10^8)^2 - (10^8)^2) + 4200^4 \cdot (10^8)^2}{6000^4 \cdot 4 \cdot (6.95 \cdot 10^8)^2} = 0.99607$$

or

$$\frac{P_{sun} - P_{with\,spot}}{P_{sun}} = 1 - \frac{P_{with\,spot}}{P_{sun}} = 1 - 0.99607 = 0.4 \text{ \%}$$

5.

a)

$$f_{day} = \frac{0.3}{0.9} + 6 = 6.3\,\text{MHz}$$

$$f_{night} = \frac{0.1}{0.9} + 4 = 4.1\,\text{MHz}$$

$$\left(2\pi f_{pe}\right)^2 = \frac{n_e e^2}{\varepsilon_0 m_e} \quad \Rightarrow$$

$$n_e = \varepsilon_0 m_e \left(\frac{2\pi f_{pe}}{e}\right)^2 = 0.0124\, f_{pe}^2$$

$$n_{e,day} = 4.9 \cdot 10^{11}\,\text{m}^{-3}$$

$$n_{e,night} = 2.1 \cdot 10^{11}\,\text{m}^{-3}$$

b)

$\Delta t = 2\,\text{h} = 7200\ \text{s}$

<u>Chapman layer: (See Tutorial 2, Problem 4)</u>

$$\frac{dn_e}{dt} = q - \alpha n_e^2$$

$$q = 0 \quad \Rightarrow$$

$$\frac{dn_e}{dt} = -\alpha n_e^2 \qquad \Rightarrow$$

$$\int \frac{dn_e}{n_e^2} = -\alpha \int dt \qquad \Rightarrow$$

$$-\frac{1}{n_e} = -\alpha t + C \qquad \Rightarrow$$

$$\alpha t = \frac{1}{n_e} + C$$

Determine C:

$$n_e(t=0) \equiv n_{e0} \quad \Rightarrow$$

$$C = -\frac{1}{n_{e0}} \quad \Rightarrow$$

$$\alpha t = \frac{1}{n_e} - \frac{1}{n_{e0}} \quad \Rightarrow$$

$$n_e = \frac{1}{\frac{1}{n_{e0}} + \alpha t} = \frac{n_{e0}}{1 + n_{e0}\alpha t} = \frac{4.9\cdot 10^{11}}{1 + 3\cdot 10^{-14}\cdot 4.9\cdot 10^{11}\cdot 7200} = 4.6\cdot 10^{9}\,\text{m}^{-3}$$

<u>Bradbury layer:</u>

With $q = 0$, we get

$$\frac{dn_e(t)}{dt} = -\beta n_e(t) \quad \Rightarrow$$

$$\frac{dn_e}{n_e} = -\beta dt \quad \Rightarrow$$

$$\ln(n_e) + C = -\beta t$$

Let us rename the constant C to $-\ln(n_{e0})$. Then

$$\ln(n_e) - \ln(n_{e0}) = -\beta t \quad \Rightarrow$$

$$\ln\left(\frac{n_e}{n_{e0}}\right) = -\beta t \quad \Rightarrow$$

$$n_e = n_{e0} e^{-\beta t} = 4.9 \cdot 10^{11} \cdot e^{-\left(10^{-4} \cdot 7200\right)} = 2.4 \cdot 10^{11} \mathrm{m}^{-3}$$

<u>Conclusion:</u>

The Bradbury layer is the more realistic model, which reflects that atomic oxygen dominates over molecular oxygen at the altitude of the F2 region.

Solutions, Examination EF2240, 2011-10-21

1. a)

v = 6/11·500 m/s = 273 m/s

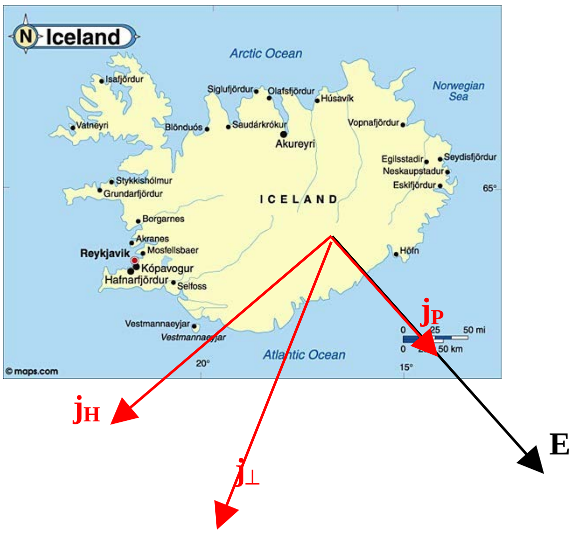
$$B = \sqrt{B_r^2 + B_\theta^2} = \sqrt{B_P^2\left(\frac{R_E}{r}\right)^6 \cos^2\theta + \left(\frac{B_P}{2}\right)^2\left(\frac{R_E}{r}\right)^6 \sin^2\theta} =$$

$$B_P\left(\frac{R_E}{r}\right)^3\sqrt{\cos^2\theta + \frac{\sin^2\theta}{4}} = B_P\left(\frac{R_E}{R_E + 300\text{ km}}\right)^3\sqrt{\cos^2 25^\circ + \frac{\sin^2 25^\circ}{4}} =$$

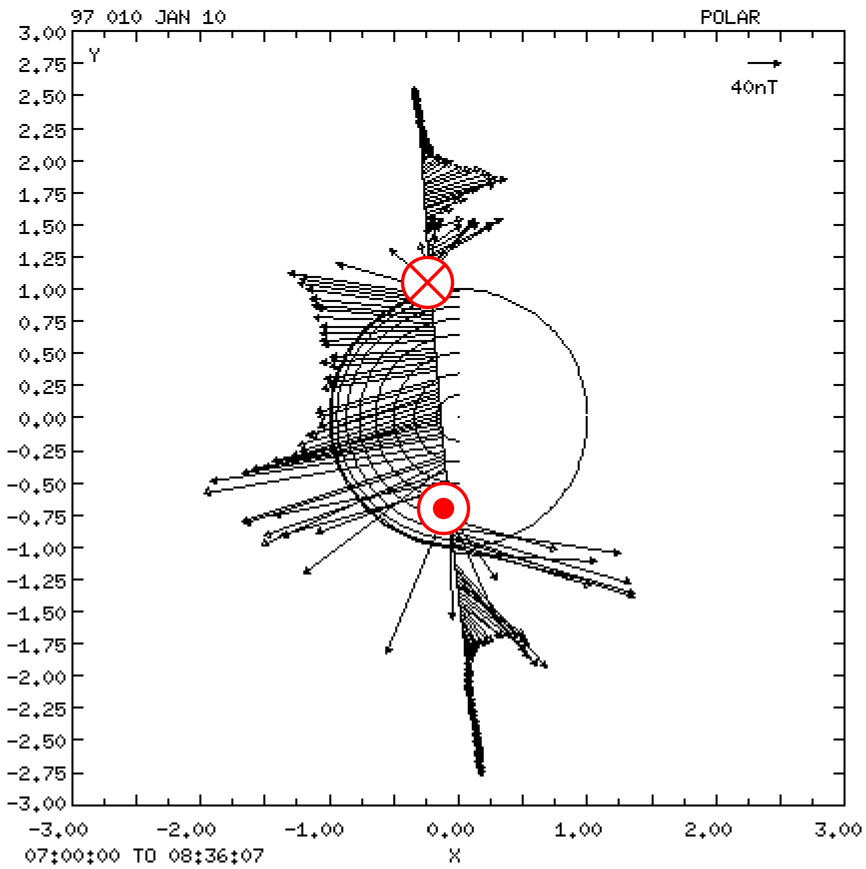
= 50 266 nT.
Then
$E = vB$ = 13.7 mV/m.

b)

Using solar maximum values at 100 km altitude, I get (night side values)
$\sigma_P = 8\cdot10^{-7}$ S/m
$\sigma_H = 7\cdot10^{-6}$ S/m
Then
$j_P = \sigma_P E = 7.3\cdot10^{-9}$ A/m^2
$j_H = \sigma_H E = 6.4\cdot10^{-8}$ A/m^2



2. a)



b)

For the upward current, e.g., we have

$$\left|j_{\parallel}\right|=\frac{1}{\mu_0}\frac{\partial B_x}{\partial y}\approx\frac{1}{\mu_0}\frac{(200+200)\cdot10^{-9}}{0.3\cdot6378\cdot10^{3}}=0.17\cdot10^{-6}\ \text{Am}^{-2}$$

3. a)

Using the scale of the sun, I estimate the radius of the CME to be

$$r_{CME}\frac{23}{3}r_{sun}=5.3\cdot10^{9}\ \text{m}$$

From the plasma frequency, we get the number density:

$$n_e=\varepsilon_0 m_e\left(\frac{2\pi f_{pe}}{e}\right)^2$$

Assuming that the CME contains only of hydrogen ions, we get the mass density

$$\rho=n_e m_p$$

The total kinetic energy of the CME is then

$$\frac{mv^2}{2}=\rho\frac{4\pi r_{CME}^3}{3}\cdot\frac{v_{CME}^2}{2}=n_e m_p\frac{2\pi r_{CME}^3 v_{CME}^2}{3}=\varepsilon_0 m_e\left(\frac{2\pi f_{pe}}{e}\right)^2 m_p\frac{2\pi r_{CME}^3 v_{CME}^2}{3}=1.3\cdot10^{23}\,\text{J}$$

b)

Evaluate the magnetic Reynolds number:

$R_m = \mu_0 \sigma l_c v_c$

We can use r_{CME} as the typical length scale, and v_{CME} as the typical velocity. Using a temperature of $2 \cdot 10^6$ K, we can evaluate the conductivity, remembering that the temperature should be given in eV. We get the conversion from

$$W = \frac{3}{2} k_B T$$

which gives the result that 1 eV corresponds to a temperature of 7729 K. We then get T = 259 eV, and

$\sigma = 7.9 \cdot 10^6$ S/m

Putting in the numbers I get

$R_m = 6.3 \cdot 10^{16} >> 1$

c)

Then the kinetic energy density is

$$\frac{1.3 \cdot 10^{23}}{4\pi r_{CME}^3 / 3} = 2.1 \cdot 10^{-7} \text{ Jm}^{-3}$$

From the gyro frequency, we get the magnetic field strength:

$$B = \frac{2\pi f_{ce} m_e}{e} = 6.1 \cdot 10^{-8} \text{ T}$$

The magnetic energy density is then

$$\frac{B^2}{2\mu_0} = 1.5 \cdot 10^{-9} \text{ T}$$

The ratio between the kinetic and magnetic energy densities is approximately 140, thus the plasma motion determines the magnetic field configuration, and not the other way around.

4. a)

Pressure balance between kinetic and magnetic pressure gives

$$\rho_{SW} v_{SW}^2 = \frac{B^2}{2\mu_0}$$

For a dipole field:

$$B^2 = B_r^2 + B_\theta^2 = (\frac{\mu_0 a}{2\pi}\frac{1}{r^3}\cos\theta)^2 + (\frac{\mu_0 a}{4\pi}\frac{1}{r^3}\sin\theta)^2$$

In the equatorial plane $\theta = 90°$, and we get

$$B^2 = (\frac{\mu_0 a}{4\pi}\frac{1}{r^3})^2$$

If we assume that the solar wind contains only protons

$\rho = n_e m_p$

and the pressure balance becomes

$$n_e m_p v^2 = \frac{\mu_0^2 a^2}{16\pi^2}\frac{1}{r^6}\frac{1}{2\mu_0}$$

Letting the standoff distance be the Mercury radius r_M, we can solve for the velocity

$$v = \sqrt{\frac{\mu_0 a^2}{n_e m_p 32\pi^2 r_M^6}} = 504 \text{ km/s.}$$

b)

For Earth, the magnetic dipole moment is $8\cdot10^{22}$ Am^2(Fälthammar p 85), and we can use a typical solar wind electron density of 8 cm^{-3}.

$$v = \sqrt{\frac{\mu_0 a_{Earth}^2}{n_e m_p 32\pi^2 r_{Earth}^6}} = 1.7\cdot10^8 \text{ m/s} = 17\,000 \text{ km/s}$$

which is totally unrealistic.

c)

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d)

The standoff distance is

$$r = \left(\frac{\mu_0 a^2}{32\pi^2 n_{e,SW} m_p v_{SW}^2}\right)^{1/6}$$

For a solar wind velocity of 300 km/s we get a standoff distance of 2901 km.
Here the standoff distance is so small compared to the Mercury radius that we should really equate the gyro radius of the proton with $r - r_M = 461$ km $=\Delta r$

$$r = \frac{m_p v}{eB}$$

We estimate the magnetic field to be constant with value B_0, from **c)**. Then

$$v = \frac{eBr}{m_p} = \frac{ev_{SW}\sqrt{2\mu_0 m_p n_{e,SW}}\, r}{m_p} = ev_{SW} r\sqrt{\frac{2\mu_0 n_{e,SW}}{m_p}} = 5.4\cdot 10^6 \text{ ms}^{-1}$$

which gives a kinetic energy of

$$\frac{m_p v^2}{2} = 2.5\cdot 10^{-14} \text{ J} = 0.15 \text{ MeV}.$$

5.

With FUV the flux of photons per unit area, the Strömgren radius is

$$r_S = \left(\frac{3N_{UV}}{4\pi\alpha_H n_H^2}\right)^{\frac{1}{3}} = \left(\frac{3\cdot 4\pi r_{star}^2 F_{UV}}{4\pi\cdot\alpha_H n_H^2}\right)^{\frac{1}{3}} \quad\Rightarrow$$

$$r_{star} = \left(\frac{r_s^3 \alpha_H n_H^2}{3F_{UV}}\right)^{\frac{1}{2}}$$

The temperature of 8000 K gives a recombination coefficient of

$\alpha_H = 2.4\cdot 10^{-19}$ m^{3}s^{-1}. Then

$r_{star} = 1.3\cdot 10^8$ m

Solutions EF2240 Exam Oct. 2012

(1.)

a) $\tan\psi = \dfrac{\omega_{sun} \cdot r}{u_{sw}}$

$r = 1 \text{ A.U.} = 1.5 \cdot 10^{11}$ m

$\omega_{sun} = \dfrac{2\pi}{T} = \dfrac{2\pi}{27 \cdot 24 \cdot 3600} = 2.7 \cdot 10^{-6}\ s^{-1}$

$u_{min} = 250 \cdot 10^{3}$ m/s $\Rightarrow$

$\psi = \arctan\left(\dfrac{2.7 \cdot 10^{-6} \cdot 1.5 \cdot 10^{11}}{250 \cdot 10^{3}}\right) = 58.3°$

$u_{max} = \arctan\left(\dfrac{2.7 \cdot 10^{-6} \cdot 1.5 \cdot 10^{11}}{750 \cdot 10^{3}}\right) = 28.4°$

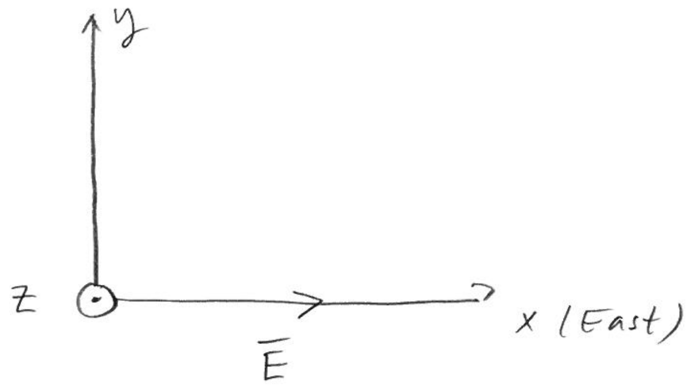
b) $W_k = \frac{\rho u_{sw}^2}{2} = \frac{m_p n_p u_{sw}^2}{2} = \frac{1.67 \cdot 10^{-27} \cdot 8 \cdot 10^6 \cdot (400 \cdot 10^3)^2}{2}$

$= 1.1 \cdot 10^{-9}$ J/m^3

$W_B = \frac{B^2}{2\mu_0} = \frac{(5 \cdot 10^{-9})^2}{2 \cdot 4\pi \cdot 10^{-7}} = 9.9 \cdot 10^{-12}$ J/m^3

$\frac{W_k}{W_B} = 111$.

(2) a)

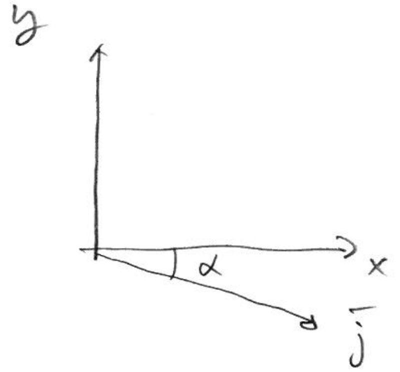


$$\bar{B} = -B\hat{z}$$

$$\bar{E} = E\hat{x}$$

$$\bar{j} = \sigma_p \bar{E} + \sigma_H \frac{\bar{B}\times\bar{E}}{B} = \sigma_p E\hat{x} + \sigma_H \frac{(-B)\hat{z}\times(E\hat{x})}{B} =$$

$$= \sigma_p E\hat{x} - \sigma_H E\hat{y}$$



$$\tan\alpha = \frac{\sigma_H E}{\sigma_p E} = \frac{\sigma_H}{\sigma_p} \doteq \frac{2\cdot 10^{-6}}{4\cdot 10^{-5}} \Rightarrow$$

Using solar max values from Fälthammar.

$$\alpha = 2.9^\circ$$

$$|\vec{j}| = \sqrt{(\sigma_P E)^2 + (\sigma_H E)^2} = E\sqrt{\sigma_P^2 + \sigma_H^2} =$$

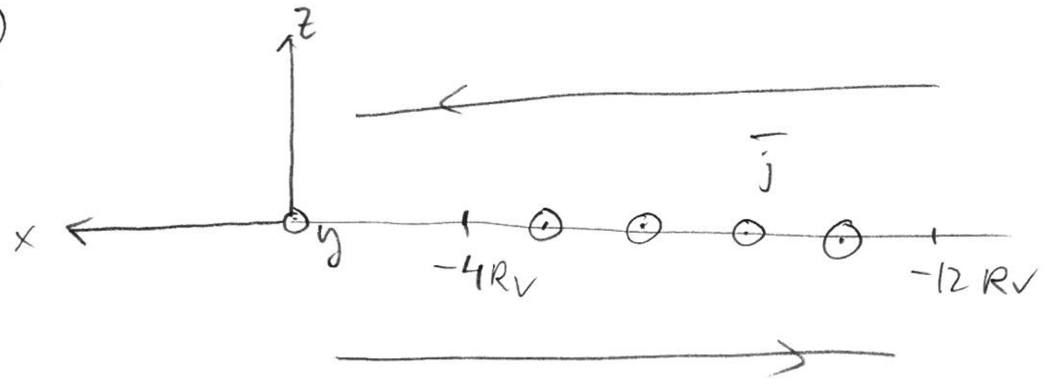
$$= 8 \cdot 10^{-3} \sqrt{(2 \cdot 10^{-6})^2 + (4 \cdot 10^{-5})^2} = 3.2 \cdot 10^{-7} \text{ A/m}^2$$

b) $\alpha = 45° \Rightarrow \sigma_P = \sigma_H$

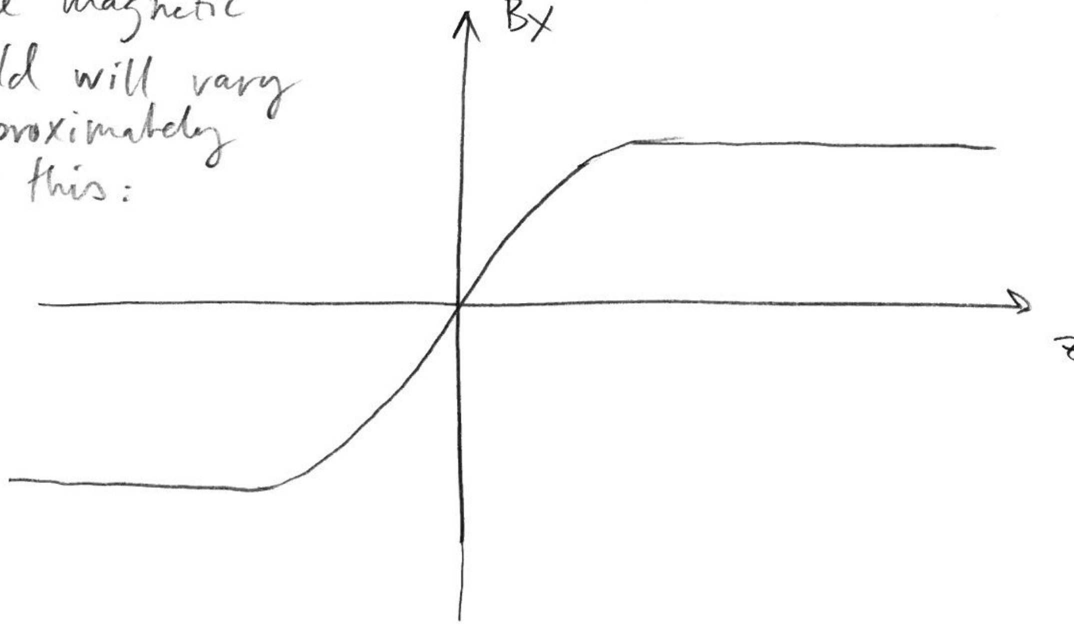
Visual inspection of Fälthammar Fig 3.2.4b-c gives

$h \approx 100$ km.

③



The magnetic field will vary approximately as this:



$$\mu_0 \bar{j} = \nabla \times \bar{B} \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}, \frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}, \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right).$$

Assume infinite current sheet extending in x- and y- directions. $\Rightarrow$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0 \Rightarrow \left[\bar{B} = (B_x, 0, 0) \right] \Rightarrow$$

$$\mu_0 \bar{j} = \left(0, \frac{\partial B_x}{\partial z}, 0 \right) \Rightarrow$$

$$\mu_0 j_y = \frac{\partial B_x}{\partial z}$$

Then the total current is

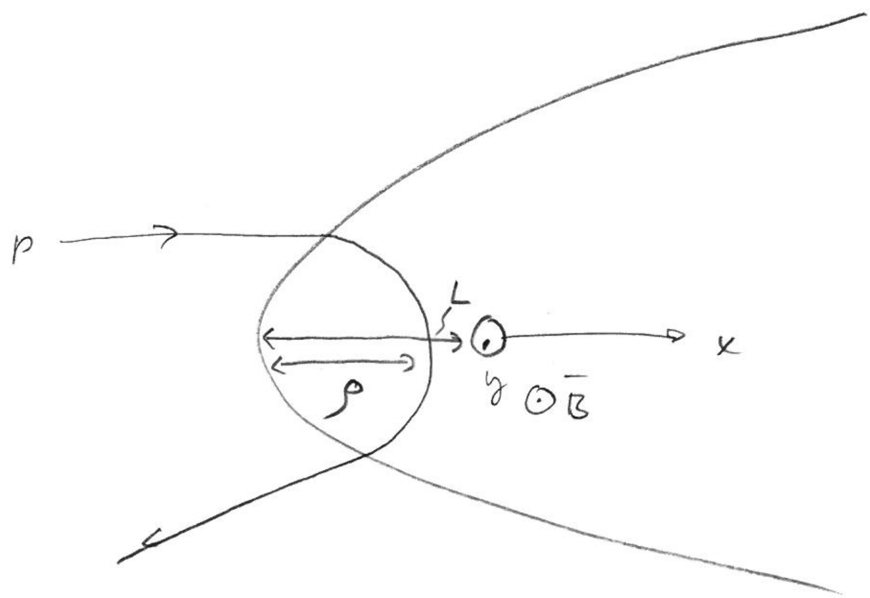
$$I = \int_{z_0}^{z_1}\int_{x_0}^{x_1} j_y \, dx \, dz = \frac{1}{\mu_0} \int_{z_0}^{z_1}\int_{x_0}^{x_1} \frac{\partial B_x}{\partial x} dx \, dz =$$

$$= \frac{1}{\mu_0} \left[B_x(z) \right]_{z_0}^{z_1} \cdot \left[x \right]_{x_0}^{x_1} = \frac{1}{\mu_0} \left(B_x(z_1) - B_x(z_0) \right) \left(x_1 - x_0 \right)$$

$$= \frac{1}{\mu_0} \left(15 \cdot 10^{-9} - (-15 \cdot 10^{-9}) \right) \left(-4 - (-12) \right) R_V =$$

$$= \frac{1}{4\pi \cdot 10^{-7}} \cdot 30 \cdot 10^{-9} \cdot 8 \cdot 6052 \cdot 10^{3} \text{ A} = 1.16 \text{ MA}$$

④



$L \approx \frac{11}{19} \cdot 100$ A.U $=$ 58 A.U (Estimated from figure 4)

The critical energy is calculated by equating L and ρ:

$$\rho = \frac{p_\perp}{qB} = L.$$

For higher energies, ρ is greater than L, and the particle will penetrate:

$\rho > L \Rightarrow$

$p_\perp > qBL \Rightarrow$

$p_\perp^2 > q^2B^2L^2$.

Now

$$E^2 = p^2c^2 + m^2c^4 \implies$$

$$p^2 = \frac{E^2 - m^2c^4}{c^2} \implies \text{(Drop the "⊥"-sign)}$$

$$\frac{E^2 - m^2c^4}{c^2} > q^2B^2L^2 \implies$$

$$E^2 > q^2B^2L^2c^2 + m^2c^4 \implies$$

$$E > \sqrt{q^2B^2L^2c^2 + m^2c^4} =$$

$$= \sqrt{\underbrace{(1.6\cdot10^{-19})^2\cdot(0.01\cdot10^{-9})^2\cdot58^2\cdot(1.5\cdot10^{11})^2\cdot(3\cdot10^8)^2}_{1.7\cdot10^{-17}} + \ldots}$$

$$\ldots + \underbrace{(1.67\cdot10^{-27})^2\cdot(3\cdot10^8)^4}_{2.3\cdot10^{-20}}$$

$$= 4.1\cdot10^{-9}\ \text{J} = 2.6\cdot10^{10}\ \text{eV} = 26\ \text{GeV}$$

5 a) The loss cone is given by

$$\sin\alpha = \sqrt{\frac{B_a}{B_{surf}}} \Rightarrow$$

$$\sin^2\alpha = \frac{B_a}{B_{surf}}$$

The magnetic field strength is given by

$$B_{surf}^2 = \underbrace{\frac{\mu_0^2 a^2}{4\pi^2 R_m^6}}_{X^2} \left(\cos^2\theta + \frac{1}{4}\sin^2\theta\right) =$$

$$= X^2\left(1 - \sin^2\theta + \frac{1}{4}\sin^2\theta\right) = X\left(1 - \frac{3}{4}\sin^2\theta\right)$$

Then

$$B_{surf} = X\sqrt{1 - \frac{3}{4}\sin^2\theta} \Rightarrow$$

$$B_{surf} = \frac{B_b}{\sin^2\alpha} \Rightarrow$$

Error: Bb should be Ba. Sorry!

$$X\sqrt{1 - \frac{3}{4}\sin^2\theta} = \frac{B_b}{\sin^2\alpha} \Rightarrow$$

$$1-\frac{3}{4}\sin^2\theta = \frac{B_b^2}{X^2\sin^4\alpha} \Rightarrow$$

$$\sin\theta = \frac{2}{\sqrt{3}}\cdot\sqrt{1-\frac{B_b^2}{X^2\sin^4\alpha}}$$

$$X = \sqrt{\frac{4\pi\cdot 10^{-7}\cdot(3\cdot 10^{19})^2}{4\pi^2(2440\cdot 10^3)^6}} = 413\ nT \Rightarrow$$

$$\sin\theta = \frac{2}{\sqrt{3}}\sqrt{1-\frac{11^2}{413^2}\cdot\frac{1}{\sin^4 10^\circ}} =$$

$$= \frac{2}{\sqrt{3}}\sqrt{1-0.78} = 0.541 \Rightarrow$$

$$\theta = 33^\circ$$

b) $$B = \left[\left(\frac{\mu_0 a}{2\pi} \frac{1}{r^3} \cos\theta \right)^2 + \left(\frac{\mu_0 a}{2\pi} \frac{1}{r^3} \frac{1}{2} \sin\theta \right)^2 \right]^{1/2}$$

$$= \frac{\mu_0 a}{2\pi} \frac{1}{r^3} \left(\cos^2\theta + \frac{1}{4} \sin^2\theta \right)^{1/2}$$

$$\frac{\partial B}{\partial \theta} = \frac{\mu_0 a}{2\pi r^3} \cdot \frac{1}{2} \cdot \left(\cos^2\theta + \frac{1}{4} \sin^2\theta \right)^{-1/2} \left(2\cos\theta(-\sin\theta) + \frac{1}{2} \sin\theta\cos\theta \right)$$

$$\theta = \frac{\pi}{2} \Rightarrow \frac{\partial B}{\partial \theta} = 0$$

$$\frac{\partial B}{\partial r} = \frac{\mu_0 a}{2\pi} (-3) r^{-4} \left(\cos^2\theta + \frac{1}{4} \sin^2\theta \right)^{1/2}$$

$$\theta = \frac{\pi}{2} \Rightarrow$$

$$\frac{\partial B}{\partial r} = \frac{-3\mu_0 a}{2\pi r^4} \left(0 + \frac{1}{4} \right)^{1/2} = \frac{-3\mu_0 a}{4\pi r^4} \Rightarrow$$

$$\nabla B = \frac{-3\mu_0 a}{4\pi r^4} \hat{r}$$

Also $\theta = \pi/2 \Rightarrow$

$$\bar{B} = \frac{\mu_0 a}{2\pi r^3} \cdot \frac{1}{2} \hat{\theta} = \frac{\mu_0 a}{4\pi r^3} \hat{\theta}$$

$$\bar{V} = -\mu \frac{(\nabla B) \times \bar{B}}{q B^2}$$

$$\mu = \frac{m v_\perp^2}{2B} = \frac{W_\perp}{B}$$

$$|V| = \frac{\mu |\nabla B|}{q B} = \frac{W_\perp |\nabla B|}{B q B} = \frac{W_\perp |\nabla B|}{q B^2} =$$

$$= \frac{W_\perp 3 \mu_0 a \, 4^2 \pi^2 r^6}{4\pi r^4 q \mu_0^2 a^2} = \frac{12\pi W_\perp r^2}{\mu_0 q a} = \frac{12\pi (R_M)^2}{\mu_0 a} \frac{W_\perp}{q} =$$

$$= 95 \frac{W_\perp}{q} = \frac{95 \cdot 10^3 q}{q} = 95 \text{ km/s}$$

$$|V_{E\times B}| = \frac{E}{B} = \frac{E 4\pi r^3}{\mu_0 a} = \frac{0.5 \cdot 10^{-3} \cdot 4\pi (2440 \cdot 10^3)^3 \cdot 4^3}{4\pi \cdot 10^{-7} \cdot 3 \cdot 10^{19}} =$$

$$= 155 \text{ km/s}$$

$$\frac{V_{\nabla B}}{V_{E\times B}} = \frac{95}{155} = 0.61$$